Vector integral calculus pdf

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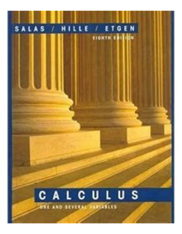


1. $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right)=x^n, n\neq -1 \Rightarrow \int x^n dx=\frac{x^{n+1}}{n+1}+C, n\neq -1$

2. $\frac{d}{dx}(\log_e x)=\frac{1}{x}\Rightarrow \int \frac{1}{x}dx=\log_e|x|+C$

3. $\frac{d}{dx}(e^x)=e^x \Rightarrow \int e^x dx=e^x+C$

4. $\frac{d}{dx}\left(\frac{a^x}{\log_e a}\right)=a^x, a>0, a\neq 1 \Rightarrow \int a^x dx=\frac{a^x}{\log_e a}+C$

5. $\frac{d}{dx}(-\cos x)=\sin x \Rightarrow \int \sin x\,dx=-\cos x+C$

6. $\frac{d}{dx}(\sin x)=\cos x \Rightarrow \int \cos x\,dx=\sin x+C$

7. $\frac{d}{dx}(\tan x)=\sec^2 x \Rightarrow \int \sec^2 x\,dx=\tan x+C$

8. $\frac{d}{dx}(-\cot x)=\mathrm{cosec}^2 x \Rightarrow \int \mathrm{cosec}^2 x\,dx=-\cot x+C$

9. $\frac{d}{dx}(\sec x)=\sec x\tan x \Rightarrow \int \sec x\tan x\,dx=\sec x+C$

10. $\frac{d}{dx}(-\mathrm{cosec}\, x)=\mathrm{cosec}\, x\cot x \Rightarrow \int \mathrm{cosec}\, x\cot x\,dx=-\mathrm{cosec}\, x+C$

11. $\frac{d}{dx}(\log\sin x)=\cot x \Rightarrow \int \cot x\,dx=\log|\sin x|+C$

12. $\frac{d}{dx}(-\log\cos x)=\tan x \Rightarrow \int \tan x\,dx=-\log|\cos x|+C$

13. $\frac{d}{dx}[\log(\sec x+\tan x)]=\sec x \Rightarrow \int \sec x\,dx=\log|\sec x+\tan x|+C$

14. $\frac{d}{dx}[\log(\mathrm{cosec}\, x-\cot x)]=\mathrm{cosec}\, x$

$\Rightarrow \int \mathrm{cosec}\, x\,dx=\log|\mathrm{cosec}\, x-\cot x|+C$

15. $\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right)=\frac{1}{x\sqrt{a^2-x^2}} \Rightarrow \int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+C$

16. $\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right)=\int\frac{-1}{\sqrt{a^2-x^2}} \Rightarrow \int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+C$

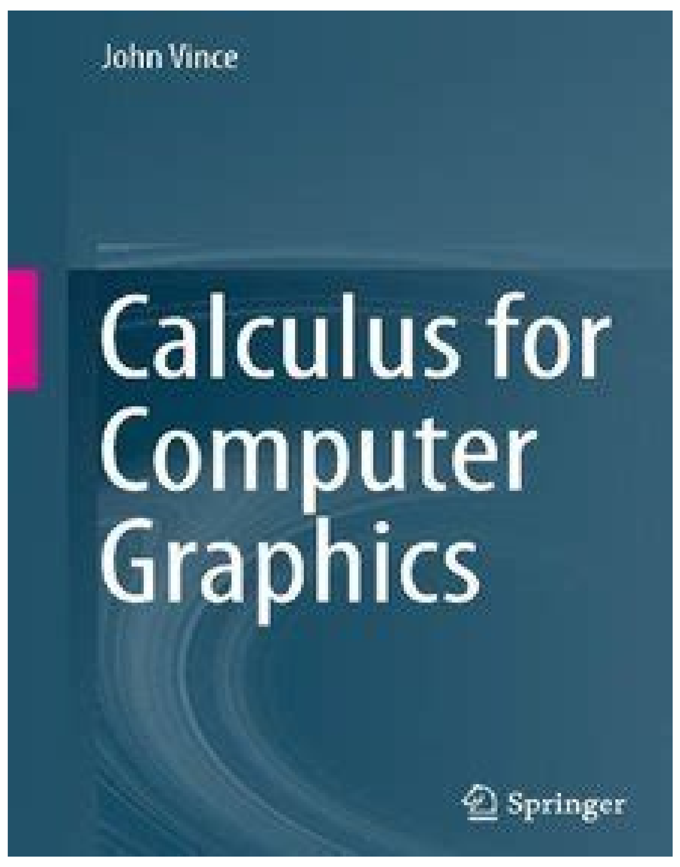
17. $\frac{d}{dx}\left(\frac{1}{a}\tan^{-1}\frac{x}{a}\right)=\frac{1}{a^2+x^2} \Rightarrow \int \frac{1}{a^2+x^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

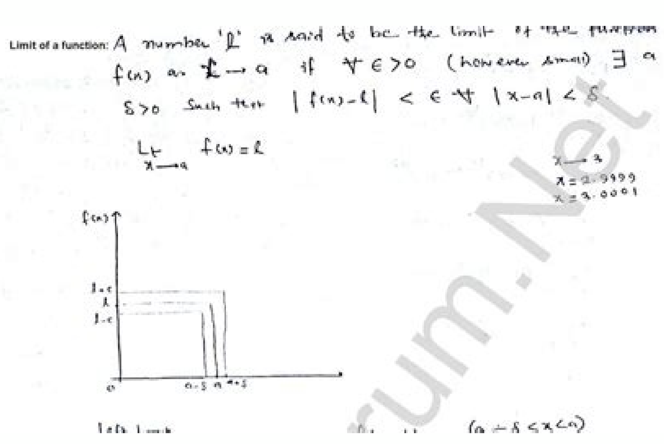
18. $\frac{d}{dx}\left(\frac{1}{a}\cot^{-1}\frac{x}{a}\right)=\frac{-1}{a^2+x^2} \Rightarrow \int \frac{-1}{a^2+x^2}dx\frac{1}{a}=\cot^{-1}\left(\frac{x}{a}\right)+C$

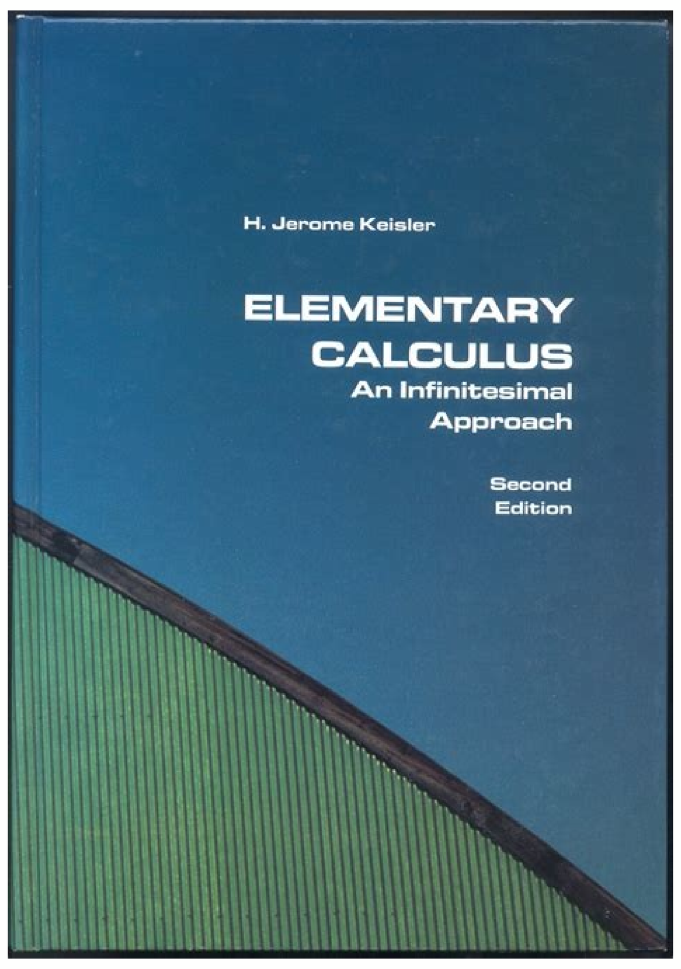
19. $\frac{d}{dx}\left(\frac{1}{a}\sec^{-1}\frac{x}{a}\right)=\frac{1}{x\sqrt{x^2-a^2}} \Rightarrow \int \frac{1}{x\sqrt{x^2-a^2}}dx=\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$

20. $\frac{d}{dx}\left(\frac{1}{a}\mathrm{cosec}^{-1}\frac{x}{a}\right)=\frac{-1}{x\sqrt{x^2-a^2}}$

$\Rightarrow \quad \int \frac{-1}{x\sqrt{x^2-a^2}}dx=\frac{1}{a}\mathrm{cosec}^{-1}\left(\frac{x}{a}\right)+C$







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Here are a set of practice problems for the Applications of Derivatives chapter of the Calculus I notes. If you'd like a pdf document containing the solutions the download tab above contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem. Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section. Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section. Rates of Change – In this section we review the main application/interpretation of derivatives from the previous chapter (i.e. rates of change) that we will be using in many of the applications in this chapter. Critical Points – In this section we give the definition of critical points. Critical points will show up in most of the sections in this chapter, so it will be important to understand them and how to find them. We will work a number of examples illustrating how to find them for a wide variety of functions. Minimum and Maximum Values – In this section we define absolute (or global) minimum and maximum values of a function and relative (or local) minimum and maximum values of a function. It is important to understand the difference between the two types of minimum/maximum (collectively called extrema) values for many of the applications in this chapter and so we use a variety of examples to help with this. We also give the Extreme Value Theorem and Fermat's Theorem, both of which are very important in the many of the applications we'll see in this chapter. Finding Absolute Extrema – In this section we discuss how to find the absolute (or global) minimum and maximum values of a function. In other words, we will be finding the largest and smallest values that a function will have. The Shape of a Graph, Part I – In this section we will discuss what the first derivative of a function can tell us about the graph of a function. The first derivative will allow us to identify the relative (or local) minimum and maximum values of a function and where a function will be increasing and decreasing. We will also give the First Derivative test which will allow us to classify critical points as relative minimums, relative maximums or neither a minimum or a maximum. The Shape of a Graph, Part II – In this section we will discuss what the second derivative of a function can tell us about the graph of a function. The second derivative will allow us to determine where the graph of a function is concave up and concave down. The second derivative will also allow us to identify any inflection points (i.e. where concavity changes) that a function may have. We will also give the Second Derivative Test that will give an alternative method for identifying some critical points (but not all) as relative minimums or relative maximums. The Mean Value Theorem – In this section we will give Rolle's Theorem and the Mean Value Theorem. With the Mean Value Theorem we will prove a couple of very nice facts, one of which will be very useful in the next chapter. Optimization Problems – In this section we will be determining the absolute minimum and/or maximum of a function that depends on two variables given some constraint, or relationship, that the two variables must always satisfy. We will discuss several methods for determining the absolute minimum or maximum of the function. Examples in this section tend to center around geometric objects such as squares, boxes, cylinders, etc. More Optimization Problems – In this section we will continue working optimization problems. The examples in this section tend to be a little more involved and will often involve situations that will be more easily described with a sketch as opposed to the 'simple' geometric objects we looked at in the previous section. L'Hospital's Rule and Indeterminate Forms – In this section we will revisit indeterminate forms and limits and take a look at L'Hospital's Rule. L'Hospital's Rule will allow us to evaluate some limits we were not able to previously. Linear Approximations – In this section we discuss using the derivative to compute a linear approximation to a function. We can use the linear approximation to a function to approximate values of the function at certain points. While it might not seem like a useful thing to do with when we have the function there really are reasons that one might want to do this. We give two ways this can be useful in the examples. Differentials – In this section we will compute the differential for a function. We will give an application of differentials in this section. However, one of the more important uses of differentials will come in the next chapter and unfortunately we will not be able to discuss it until then. Newton's Method – In this section we will discuss Newton's Method. Newton's Method is an application of derivatives that will allow us to approximate solutions to an equation. There are many equations that cannot be solved directly and with this method we can get approximations to the solutions to many of those equations. Business Applications – In this section we will give a cursory discussion of some basic applications of derivatives to the business field. We will revisit finding the maximum and/or minimum function value and we will define the marginal cost function, the average cost, the revenue function, the marginal revenue function and the marginal profit function. Note that this section is only intended to introduce these concepts and not teach you everything about them. Calculus of vector-valued functions Not to be confused with Geometric calculus or Matrix calculus. This article includes a list of general references, but it lacks sufficient corresponding inline citations. Please help to improve this article by introducing more precise citations. (February 2016) (Learn how and when to remove this template message) Part of a series of articles aboutCalculus Fundamental theorem Leibniz integral rule Limits of functions Continuity Mean value theorem Rolle's theorem Differential Definitions Derivative (generalizations) Differential infinitesimal of a function total Concepts Differentiation notation Second derivative Implicit differentiation Logarithmic differentiation Related rates Taylor's theorem Rules and identities Sum Product Chain Power Quotient L'Hôpital's rule Inverse General Leibniz Faà di Bruno's formula Reynolds Integral Lists of integrals Integral transform Definitions Antiderivative Integral (improper) Riemann integral Lebesgue integration Contour integration Integral of inverse functions Integration by Parts Discs Cylindrical shells Substitution (trigonometric, Weierstrass, Euler) Euler's formula Partial fractions Changing order Reduction formulae Differentiating under the integral sign Risch algorithm Series Geometric (arithmetico-geometric) Harmonic Alternating Power Binomial Taylor Convergence tests Summand limit (term test) Ratio Root Integral Direct comparison Limit comparison Alternating series Cauchy condensation Dirichlet Abel Vector Gradient Divergence Curl Laplacian Directional derivative Identities Theorems Gradient Green's Stokes' Divergence generalized Stokes Multivariable Formalisms Matrix Tensor Exterior Geometric Definitions Partial derivative Multiple integral Line integral Surface integral Volume integral Jacobian Hessian Advanced Calculus on Euclidean space Limit of distributions Specialized Fractional Malliavin Stochastic Variations Miscellaneous Precalculus History Glossary List of topics Integration Bee Analysis vte Vector calculus, or vector analysis, is concerned with differentiation and integration of vector fields, primarily in 3-dimensional Euclidean space R 3 . {\displaystyle \mathbb {R} ^{3}.} The term "vector calculus" is sometimes used as a synonym for the broader subject of multivariable calculus, which spans vector calculus as well as partial differentiation and multiple integration. Vector calculus plays an important role in differential geometry and in the study of partial differential equations. It is used extensively in physics and engineering, especially in the description of electromagnetic fields, gravitational fields, and fluid flow. Vector calculus was developed from quaternion analysis by J. Willard Gibbs and Oliver Heaviside near the end of the 19th century, and most of the notation and terminology was established by Gibbs and Edwin Bidwell Wilson in their 1901 book, Vector Analysis. In the conventional form using cross products, vector calculus does not generalize to higher dimensions, while the alternative approach of geometric algebra which uses exterior products does (see § Generalizations below for more). Basic objects Scalar fields Main article: Scalar field A scalar field associates a scalar value to every point in a space. The scalar is a mathematical number representing a physical quantity. Examples of scalar fields in applications include the temperature distribution throughout space, the pressure distribution in a fluid, and spin-zero quantum fields (known as scalar bosons), such as the Higgs field. These fields are the subject of scalar field theory. Vector fields Main article: Vector field A vector field is an assignment of a vector to each point in a space.[1] A vector field in the plane, for instance, can be visualized as a collection of arrows with a given magnitude and direction each attached to a point in the plane. Vector fields are often used to model, for example, the speed and direction of a moving fluid throughout space, or the strength and direction of some force, such as the magnetic or gravitational force, as it changes from point to point. This can be used, for example, to calculate work done over a line. Vectors and pseudovectors In more advanced treatments, one further distinguishes pseudovector fields and pseudoscalar fields, which are identical to vector fields and scalar fields, except that they change sign under an orientation-reversing map: for example, the curl of a vector field is a pseudovector field, and if one reflects a vector field, the curl points in the opposite direction. This distinction is clarified and elaborated in geometric algebra, as described below. Vector algebra Main article: Euclidean vector § Basic properties The algebraic (non-differential) operations in vector calculus are referred to as vector algebra, being defined for a vector space and then globally applied to a vector field. The basic algebraic operations consist of: Notations in vector calculus Operation Notation Description Vector addition v 1 + v 2 {\displaystyle \mathbf {v} _{1}+\mathbf {v} _{2}} Addition of two vectors, yielding a vector. Scalar multiplication a v {\displaystyle a\mathbf {v} } Multiplication of a scalar and a vector, yielding a vector. Dot product v 1 · v 2 {\displaystyle \mathbf {v} _{1}\cdot \mathbf {v} _{2}} Multiplication of two vectors, yielding a scalar. Cross product v 1 × v 2 {\displaystyle \mathbf {v} _{1}\times \mathbf {v} _{2}} Multiplication of two vectors in R 3 {\displaystyle \mathbb {R} ^{3}} , yielding a (pseudo)vector. Also commonly used are the two triple products: Vector calculus triple products Operation Notation Description Scalar triple product v 1 · (v 2 × v 3) {\displaystyle \mathbf {v} _{1}\cdot \left(\mathbf {v} _{2}\times \mathbf {v} _{3}\right)} The dot product of the cross product of two vectors. Vector triple product v 1 × (v 2 × v 3) {\displaystyle \mathbf {v} _{1}\times \left(\mathbf {v} _{2}\times \mathbf {v} _{3}\right)} The cross product of the cross product of two vectors. Operators and theorems Main article: Vector calculus identities Differential operators Main articles: Gradient, Divergence, Curl (mathematics), and Laplacian Vector calculus studies various differential operators defined on scalar or vector fields, which are typically expressed in terms of the del operator (∇ {\displaystyle abla }), also known as "nabla". The three basic vector operators are:[2] Differential operators in vector calculus Operation Notation Description Notationalanalogy Domain/Range Gradient grad ⁡ (f) = ∇ f {\displaystyle \operatorname {grad} (f)=abla f} Measures the rate and direction of change in a scalar field. Scalar multiplication Maps scalar fields to vector fields. Divergence div ⁡ (F) = ∇ · F {\displaystyle \operatorname {div} (\mathbf {F})=abla \cdot \mathbf {F} } Measures the scalar of a source or sink at a given point in a vector field. Dot product Maps vector fields to scalar fields. Curl curl ⁡ (F) = ∇ × F {\displaystyle \operatorname {curl} (\mathbf {F})=abla \times \mathbf {F} } Measures the tendency to rotate about a point in a vector field in R 3 {\displaystyle \mathbb {R} ^{3}} . Cross product Maps vector fields to (pseudo)vector fields. f denotes a scalar field and F denotes a vector field Also commonly used are the two Laplace operators: Laplace operators in vector calculus Operation Notation Description Domain/Range Laplacian Δ f = ∇ 2 f = ∇ · ∇ f {\displaystyle \Delta f=abla ^{2}f=abla \cdot abla f} Measures the difference between the value of the scalar field with its average on infinitesimal balls. Maps between scalar fields. Vector Laplacian ∇ 2 F = ∇ (∇ · F) − ∇ × (∇ × F) {\displaystyle abla ^{2}\mathbf {F} =abla (abla \cdot \mathbf {F})-abla \times (abla \times \mathbf {F})} Measures the difference between the value of the vector field with its average on infinitesimal balls. Maps between vector fields. f denotes a scalar field and F denotes a vector field A quantity called the Jacobian matrix is useful for studying functions when both the domain and range of the function are multivariable, such as a change of variables during integration. Integral theorems The three basic vector operators have corresponding theorems which generalize the fundamental theorem of calculus to higher dimensions: Integral theorems of vector calculus Theorem Statement Description Gradient theorem ∫ L ⊂ R n ∇ φ · d r = φ (q) − φ (p) for L = L [p → q] {\displaystyle \int _{L\subset \mathbb {R} ^{n}}\!\!\!abla \varphi \cdot d\mathbf {r} \ =\ \varphi \left(\mathbf {q} \right)-\varphi \left(\mathbf {p} \right)\ \ {\text{ for }}\ \ L=L[p\to q]} The line integral of the gradient of a scalar field over a curve L is equal to the change in the scalar field between the endpoints p and q of the curve. Divergence theorem ∫ ⋯ ∫ V ⊂ R n (∇ · F) d V = ∮ ⋯ ∮ ∂ V n − 1 F · d S {\displaystyle \underbrace {\int \!\cdots \!\int _{V\subset \mathbb {R} ^{n}}} _{n}(abla \cdot \mathbf {F})\,dV\ =\ \underbrace {\oint \!\cdots \!\oint _{\partial V}} _{n-1}\mathbf {F} \cdot d\mathbf {S} } The integral of the divergence of a vector field over an n-dimensional solid V is equal to the flux of the vector field through the (n−1)-dimensional closed boundary surface of the solid. Curl (Kelvin–Stokes) theorem ∬ Σ ⊂ R 3 (∇ × F) ⋅ d Σ = ∮ ∂ Σ F ⋅ d r {\displaystyle \iint _{\Sigma \,\subset \mathbb {R} ^{3}}(abla \times \mathbf {F})\cdot d\mathbf {\Sigma } \ =\ \oint _{\!\!\partial \Sigma }\mathbf {F} \cdot d\mathbf {r} } The integral of the curl of a vector field over a surface Σ in R 3 {\displaystyle \mathbb {R} ^{3}} is equal to the circulation of the vector field around the closed curve bounding the surface. φ {\displaystyle \varphi } denotes a scalar field and F denotes a vector field In two dimensions, the divergence and curl theorems reduce to the Green's theorem: Green's theorem of vector calculus Theorem Statement Description Green's theorem ∬ A ⊂ R 2 (∂ M ∂ x − ∂ L ∂ y) d A = ∮ ∂ A (L d x + M d y) {\displaystyle \iint _{A\,\subset \mathbb {R} ^{2}}\left({\frac {\partial M}{\partial x}}-{\frac {\partial L}{\partial y}}\right)dA\ =\ \oint _{\partial A}\left(L\,dx+M\,dy\right)} The integral of the divergence (or curl) of a vector field over some region A in R 2 {\displaystyle \mathbb {R} ^{2}} equals the flux (or circulation) of the vector field over the closed curve bounding the region. For divergence, F = (M, −L). For curl, F = (L, M, 0). L and M are functions of (x, y). Applications Linear approximations Main article: Linear approximation Linear approximations are used to replace complicated functions with linear functions that are almost the same. Given a differentiable function f(x, y) with real values, one can approximate f(x, y) for (x, y) close to (a, b) by the formula f (x , y) ≈ f (a , b) + ∂ f ∂ x (a , b) (x − a) + ∂ f ∂ y (a , b) (y − b) . {\displaystyle f(x,y)\ \approx \ f(a,b)+{\tfrac {\partial f}{\partial x}}(a,b)\,(x-a)+{\tfrac {\partial f}{\partial y}}(a,b)\,(y-b).} The right-hand side is the equation of the plane tangent to the graph of z = f(x, y) at (a, b). Optimization Main article: Mathematical optimization For a continuously differentiable function of several real variables, a point P (that is, a set of values for the input variables, which is viewed as a point in Rn) is critical if all of the partial derivatives of the function are zero at P, or, equivalently, if its gradient is zero. The critical values are the values of the function at the critical points. If the function is smooth, or, at least twice continuously differentiable, a critical point may be either a local maximum, a local minimum or a saddle point. The different cases may be distinguished by considering the eigenvalues of the Hessian matrix of second derivatives. By Fermat's theorem, all local maxima and minima of a differentiable function occur at critical points. Therefore, to find the local maxima and minima, it suffices, theoretically, to compute the zeros of the gradient and the eigenvalues of the Hessian matrix at these zeros. Physics and engineering Vector calculus is particularly useful in studying: Center of mass Field theory Kinematics Maxwell's equations Generalizations This section does not cite any sources. Please help improve this section by adding citations to reliable sources. Unsourced material may be challenged and removed. (August 2019) (Learn how and when to remove this template message) Different 3-manifolds Vector calculus is initially defined for Euclidean 3-space, R 3 , {\displaystyle \mathbb {R} ^{3},} which has additional structure beyond simply being a 3-dimensional real vector space, namely: a norm (giving a notion of length) defined via an inner product (the dot product), which in turn gives a notion of angle, and an orientation, which gives a notion of left-handed and right-handed. These structures give rise to a volume form, and also the cross product, which is used pervasively in vector calculus. The gradient and divergence require only the inner product, while the curl and the cross product also requires the handedness of the coordinate system to be taken into account (see cross product and handedness for more detail). Vector calculus can be defined on other 3-dimensional real vector spaces if they have an inner product (or more generally a symmetric nondegenerate form) and an orientation; note that this is less data than an isomorphism to Euclidean space, as it does not require a set of coordinates (a frame of reference), which reflects the fact that vector calculus is invariant under rotations (the special orthogonal group SO(3)). More generally, vector calculus can be defined on any 3-dimensional oriented Riemannian manifold, or more generally pseudo-Riemannian manifold. This structure simply means that the tangent space at each point has an inner product (more generally, a symmetric nondegenerate form) and an orientation, or more globally that there is a symmetric nondegenerate metric tensor and an orientation, and works because vector calculus is defined in terms of tangent vectors at each point. Other dimensions Most of the analytic results are easily understood, in a more general form, using the machinery of differential geometry, of which vector calculus forms a subset. Grad and div generalize immediately to other dimensions, as do the gradient theorem, divergence theorem, and Laplacian (yielding harmonic analysis), while curl and cross product do not generalize as directly. From a general point of view, the various fields in (3-dimensional) vector calculus are uniformly seen as being k-vector fields: scalar fields are 0-vector fields, vector fields are 1-vector fields, pseudovector fields are 2-vector fields, and pseudoscalar fields are 3-vector fields. In higher dimensions there are additional types of fields (scalar/vector/pseudovector/pseudoscalar corresponding to 0/1/n−1/n dimensions, which is exhaustive in dimension 3), so one cannot only work with (pseudo)scalars and (pseudo)vectors. In any dimension, assuming a nondegenerate form, grad of a scalar function is a vector field, and div of a vector field is a scalar function, but only in dimension 3 or 7[3] (and, trivially, in dimension 0 or 1) is the curl of a vector field a vector field, and only in 3 or 7 dimensions can a cross product be defined (generalizations in other dimensionalities either require n − 1 {\displaystyle n-1} vectors to yield 1 vector, or are alternative Lie algebras, which are more general antisymmetric bilinear products). The generalization of grad and div, and how curl may be generalized is elaborated at Curl: Generalizations; in brief, the curl of a vector field is a bivector field, which may be interpreted as the special orthogonal Lie algebra of infinitesimal rotations; however, this cannot be identified with a vector field because the dimensions differ – there are 3 dimensions of rotations in 3 dimensions, but 6 dimensions of rotations in 4 dimensions (and more generally (n 2) = 1 2 n (n − 1) {\displaystyle \textstyle {{\binom {n}{2}}={\frac {1}{2}}n(n-1)}} dimensions of rotations in n dimensions). There are two important alternative generalizations of vector calculus. The first, geometric algebra, uses k-vector fields instead of vector fields (in 3 or fewer dimensions, every k-vector field can be identified with a scalar function or vector field, but this is not true in higher dimensions). This replaces the cross product, which is specific to 3 dimensions, taking in two vector fields and giving as output a vector field, with the exterior product, which exists in all dimensions and takes in two vector fields, giving as output a bivector (2-vector) field. This product yields Clifford algebras as the algebraic structure on vector spaces (with an orientation and nondegenerate form). Geometric algebra is mostly used in generalizations of physics and other applied fields to higher dimensions. The second generalization uses differential forms (k-covector fields) instead of vector fields or k-vector fields, and is widely used in mathematics, particularly in differential geometry, geometric topology, and harmonic analysis, in particular yielding Hodge theory on oriented pseudo-Riemannian manifolds. From this point of view, grad, curl, and div correspond to the exterior derivative of 0-forms, 1-forms, and 2-forms, respectively, and the key theorems of vector calculus are all special cases of the general form of Stokes' theorem. From the point of view of both of these generalizations, vector calculus implicitly identifies mathematically distinct objects, which makes the presentation simpler but the underlying mathematical structure and generalizations less clear. From the point of view of geometric algebra, vector calculus implicitly identifies k-vector fields with vector fields or scalar functions: 0-vectors and 3-vectors with scalars, 1-vectors and 2-vectors with vectors. From the point of view of differential forms, vector calculus implicitly identifies k-forms with scalar fields or vector fields: 0-forms and 3-forms with scalar fields, 1-forms and 2-forms with vector fields. Thus for example the curl naturally takes as input a vector field or 1-form, but naturally has as output a 2-vector field or 2-form (hence pseudovector field), which is then interpreted as a vector field, rather than directly taking a vector field to a vector field; this is reflected in the curl of a vector field in higher dimensions not having as output a vector field. See also Analysis of vector-valued curves Real-valued function Function of a real variable Function of several real variables Vector calculus identities Vector algebra relations Del in cylindrical and spherical coordinates Directional derivative Conservative vector field Solenoidal vector field Laplacian vector field Helmholtz decomposition Orthogonal coordinates Skew coordinates Curvilinear coordinates Tensor Geometric calculus References Citations ^ Galbis, Antonio & Maestre, Manuel (2012). Vector Analysis Versus Vector Calculus. Springer. p. 12. 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